Finite Geometries

Aaron Cinzori

Fall 2012
Four-Point Geometry

Undefined Terms: Point, Line, On.
Undefined Terms: Point, Line, On.

**Axiom 1:** There exist exactly four distinct points.
Undefined Terms: Point, Line, On.

Axiom 1: There exist exactly four distinct points.

Axiom 2: Any two distinct points have exactly one line on both of them.
Four-Point Geometry

**Undefined Terms:** Point, Line, On.

**Axiom 1:** There exist exactly four distinct points.

**Axiom 2:** Any two distinct points have exactly one line on both of them.

**Axiom 3:** Each line is on exactly two points.
Undefined Terms: Point, Line, On.

**Axiom 1:** There exist exactly four distinct points.

**Axiom 2:** Any two distinct points have exactly one line on both of them.

**Axiom 3:** Each line is on exactly two points.

**Def 1.3.1:** Two lines on the same point are said to intersect.

**Def 1.3.2:** Two lines that do not intersect are called parallel.
Undefined Terms: Point, Line, On.

Axiom 1: There exist exactly four distinct points.

Axiom 2: Any two distinct points have exactly one line on both of them.

Axiom 3: Each line is on exactly two points.

Def 1.3.1: Two lines on the same point are said to intersect.

Def 1.3.2: Two lines that do not intersect are called parallel.

4-point Theorem 1: If two distinct lines intersect, then they have exactly one point in common.
Four-Point Geometry

**Undefined Terms:** Point, Line, On.

**Axiom 1:** There exist exactly four distinct points.

**Axiom 2:** Any two distinct points have exactly one line on both of them.

**Axiom 3:** Each line is on exactly two points.

**Def 1.3.1:** Two lines on the same point are said to *intersect*.

**Def 1.3.2:** Two lines that do not intersect are called *parallel*.
Four-Point Geometry

Undefined Terms: Point, Line, On.

Axiom 1: There exist exactly four distinct points.

Axiom 2: Any two distinct points have exactly one line on both of them.

Axiom 3: Each line is on exactly two points.

Def 1.3.1: Two lines on the same point are said to intersect

Def 1.3.2: Two lines that do not intersect are called parallel.

4-point Theorem 2: Four-point geometry has exactly 6 lines.
Four-Point Geometry

Undefined Terms: Point, Line, On.

**Axiom 1:** There exist exactly four distinct points.

**Axiom 2:** Any two distinct points have exactly one line on both of them.

**Axiom 3:** Each line is on exactly two points.

**Def 1.3.1:** Two lines on the same point are said to intersect.

**Def 1.3.2:** Two lines that do not intersect are called parallel.
Four-Point Geometry

Undefined Terms: Point, Line, On.

Axiom 1: There exist exactly four distinct points.

Axiom 2: Any two distinct points have exactly one line on both of them.

Axiom 3: Each line is on exactly two points.

Def 1.3.1: Two lines on the same point are said to intersect.

Def 1.3.2: Two lines that do not intersect are called parallel.

4-point Theorem 3: Each point of the four-point geometry has exactly three lines on it.
Four-Point Geometry

Undefined Terms: Point, Line, On.

Axiom 1: There exist exactly four distinct points.
Axiom 2: Any two distinct points have exactly one line on both of them.
Axiom 3: Each line is on exactly two points.
Def 1.3.1: Two lines on the same point are said to intersect
Def 1.3.2: Two lines that do not intersect are called parallel.
Undefined Terms: Point, Line, On.

Axiom 1: There exist exactly four distinct points.

Axiom 2: Any two distinct points have exactly one line on both of them.

Axiom 3: Each line is on exactly two points.

Def 1.3.1: Two lines on the same point are said to intersect.

Def 1.3.2: Two lines that do not intersect are called parallel.

4-point Theorem 4: Each distinct line has exactly one line parallel to it.
Fano’s Geometry

**Undefined Terms:** Point, Line, On.
Fano’s Geometry

Undefined Terms: Point, Line, On.

Axiom 1: There exists at least one line.
Fano’s Geometry

Undefined Terms: Point, Line, On.

Axiom 1: There exists at least one line.

Axiom 2: There exist exactly three points on every line.
Fano’s Geometry

Undefined Terms: Point, Line, On.

Axiom 1: There exists at least one line.
Axiom 2: There exist exactly three points on every line.
Axiom 3: Not all points are on the same line.
Fano’s Geometry

Undefined Terms: Point, Line, On.

Axiom 1: There exists at least one line.

Axiom 2: There exist exactly three points on every line.

Axiom 3: Not all points are on the same line.

Axiom 4: There exists exactly one line on any two distinct points.
Fano’s Geometry

Undefined Terms: Point, Line, On.

Axiom 1: There exists at least one line.
Axiom 2: There exist exactly three points on every line.
Axiom 3: Not all points are on the same line.
Axiom 4: There exists exactly one line on any two distinct points.
Axiom 5: There exists at least one point on any two distinct lines.
Fano’s Geometry

Undefined Terms: Point, Line, On.

Axiom 1: There exists at least one line.

Axiom 2: There exist exactly three points on every line.

Axiom 3: Not all points are on the same line.

Axiom 4: There exists exactly one line on any two distinct points.

Axiom 5: There exists at least one point on any two distinct lines.

Fano Theorem 1: Two distinct lines have exactly one point in common.
Fano’s Geometry

Undefined Terms: Point, Line, On.

Axiom 1: There exists at least one line.

Axiom 2: There exist exactly three points on every line.

Axiom 3: Not all points are on the same line.

Axiom 4: There exists exactly one line on any two distinct points.

Axiom 5: There exists at least one point on any two distinct lines.
Fano’s Geometry

Undefined Terms: Point, Line, On.

Axiom 1: There exists at least one line.
Axiom 2: There exist exactly three points on every line.
Axiom 3: Not all points are on the same line.
Axiom 4: There exists exactly one line on any two distinct points.
Axiom 5: There exists at least one point on any two distinct lines.

Fano Theorem 2: There are exactly seven points in Fano’s geometry.
Young’s Geometry

Undefined Terms: Point, Line, On.
Young’s Geometry

Undefined Terms: Point, Line, On.

Axiom 1: There exists at least one line.
Undefined Terms: Point, Line, On.

Axiom 1: There exists at least one line.
Axiom 2: There exist exactly three points on every line.
Young’s Geometry

Undefined Terms: Point, Line, On.

Axiom 1: There exists at least one line.
Axiom 2: There exist exactly three points on every line.
Axiom 3: Not all points are on the same line.
Young's Geometry

Undefined Terms: Point, Line, On.

Axiom 1: There exists at least one line.

Axiom 2: There exist exactly three points on every line.

Axiom 3: Not all points are on the same line.

Axiom 4: There exists exactly one line on any two distinct points.

Young's Theorem 1: Every point in Young's geometry is on at least 4 lines.
Young’s Geometry

**Undefined Terms:** Point, Line, On.

**Axiom 1:** There exists at least one line.

**Axiom 2:** There exist exactly three points on every line.

**Axiom 3:** Not all points are on the same line.

**Axiom 4:** There exists exactly one line on any two distinct points.

**Axiom 5:** For each line \( \ell \) and each point \( P \) not on \( \ell \), there exists exactly one line on \( P \) that does not contain any points on \( \ell \).
Young’s Geometry

Undefined Terms: Point, Line, On.

Axiom 1: There exists at least one line.

Axiom 2: There exist exactly three points on every line.

Axiom 3: Not all points are on the same line.

Axiom 4: There exists exactly one line on any two distinct points.

Axiom 5: For each line $\ell$ and each point $P$ not on $\ell$, there exists exactly one line on $P$ that does not contain any points on $\ell$.

Young’s Theorem 1: Every point in Young’s geometry is on at least 4 lines.
Young’s Geometry

Undefined Terms: Point, Line, On.

Axiom 1: There exists at least one line.
Axiom 2: There exist exactly three points on every line.
Axiom 3: Not all points are on the same line.
Axiom 4: There exists exactly one line on any two distinct points.
Axiom 5: For each line \( \ell \) and each point \( P \) not on \( \ell \), there exists exactly one line on \( P \) that does not contain any points on \( \ell \).
Undefined Terms: Point, Line, On.

Axiom 1: There exists at least one line.

Axiom 2: There exist exactly three points on every line.

Axiom 3: Not all points are on the same line.

Axiom 4: There exists exactly one line on any two distinct points.

Axiom 5: For each line $\ell$ and each point $P$ not on $\ell$, there exists exactly one line on $P$ that does not contain any points on $\ell$.

Young’s Theorem 2: Each line has exactly two lines parallel to it.
Young’s Geometry

Undefined Terms: Point, Line, On.

Axiom 1: There exists at least one line.
Axiom 2: There exist exactly three points on every line.
Axiom 3: Not all points are on the same line.
Axiom 4: There exists exactly one line on any two distinct points.
Axiom 5: For each line $\ell$ and each point $P$ not on $\ell$, there exists exactly one line on $P$ that does not contain any points on $\ell$. 

Young’s Theorem 3: Young’s geometry contains exactly 12 lines.
Young’s Geometry

Undefined Terms: Point, Line, On.

Axiom 1: There exists at least one line.
Axiom 2: There exist exactly three points on every line.
Axiom 3: Not all points are on the same line.
Axiom 4: There exists exactly one line on any two distinct points.
Axiom 5: For each line $\ell$ and each point $P$ not on $\ell$, there exists exactly one line on $P$ that does not contain any points on $\ell$.

Young’s Theorem 3: Young’s geometry contains exactly 12 lines.