Modelling Assessment Data with a Hierarchical Approach

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Background of study

Proposed curriculum for introductory statistics

- Inferential concepts taught through randomization tests (Rossman and Chance, 2008)
- Collection of statistical learning modules and technology tools (Rossman/Chance Applet Collection)
- Has significantly improved student gains at Hope College (Tintle et al, 2011)

Data from NSF grant funded research by Tintle et al, 2015
Data context

Data context: **Student assessment data**
- Student responses at beginning and end of course
  - Concept inventory modelled after **CAOS** (Garfield et al, 2006)
  - **SATS** attitudes survey (Schau, 2005)
- **Instructor survey**: teaching style and course background
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- **Instructor survey**: teaching style and course background
- **Multiple institutions** in Fall 2013 and Spring 2014
  - 24 distinct instructors
    - 15 from Fall 2013 and 9 from Spring 2014
  - 36 instructor-terms ("sections")
    - 15 from Fall 2013 and 21 from Spring 2014
  - 1,116 students
    - Spent at least 10 minutes on pre/post concepts with at least 40% section participation
Research goals

Primary research goal: **Model student gains from the curriculum**

- **Predictor variables**
  1. **Student characteristics**
     - e.g. scores on the 6 attitude components of SATS
  2. **Instructor characteristics**
     - e.g. experience with randomization-based curriculum
Research goals

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- Predictor variables
  - Student characteristics
    - e.g. scores on the 6 attitude components of SATS
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    - e.g. experience with randomization-based curriculum

Secondary research goals:
1. Research an appropriate modelling method
   - Hierarchical models
   - *lme4*, *nlme*, and *MCMCglmm* packages in R
2. Develop a web application focusing on the modelling method
   - *shiny* package in R
   - Provide visualizations in understanding the role of variability in hierarchical models
Why use hierarchical models?

Two main reasons

1. Nested data
   - Multiple sets of observational units (hierarchy)
   - Large number of groups
   - Independence assumption in OLS is violated

2. Random effects
   - Allow certain coefficients to vary by group
   - Error terms at different levels in the hierarchy

<table>
<thead>
<tr>
<th>Section</th>
<th>Student</th>
<th>Instructor Experience</th>
<th>Student attitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A</td>
<td>Experienced</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>B</td>
<td>Experienced</td>
<td>6</td>
</tr>
<tr>
<td>1</td>
<td>C</td>
<td>Experienced</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>D</td>
<td>New</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>E</td>
<td>New</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>F</td>
<td>New</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>G</td>
<td>Non-user</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>H</td>
<td>Non-user</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>I</td>
<td>Non-user</td>
<td>5</td>
</tr>
</tbody>
</table>
Varying-intercept model

Model

Conceptual gains = overall average + section "effect" + student "effect"

\[ y_{ij} = \mu + \eta_j + \epsilon_{ij}, \text{ for the } i\text{th student in the } j\text{th section} \]

where \( \eta_j \sim N(0, \sigma^2_\eta) \) and \( \epsilon_{ij} \sim N(0, \sigma^2_y) \)
### Varying-intercept model

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#### Hyperparameters:

- \( \hat{\mu} = 0.082 \) (overall average of conceptual gains)
- \( \hat{\sigma}^2_y = 0.014 \) (within-sections/between-student variability)
- \( \hat{\sigma}^2_\eta = 0.0004 \) (between-section variability)
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**Intraclass correlation coefficient:**

Disparity among sections (without any additional section or student characteristics) accounts for 2.8% of total variability in conceptual gains.
Varying-intercept model visualizations

Sections with *small* \( n \) (e.g. 24) get **pulled toward overall average**, while sections with *large* \( n \) (e.g. 3) **rely more on their own averages**.
Varying-intercept and -slope with level-2 predictor model

Model with "cross-level" interaction

Conceptual gains = student expected difficulty + instructor curriculum experience + expected difficulty*curriculum experience + section "effect" + student "effect"

\[ y_{ij} = \alpha_j + \beta_j x_{ij} + \epsilon_{ij} \]

\[ \alpha_j = \gamma_{\alpha 0} + \gamma_{\alpha 1} u_j + \eta_{\alpha j} \]

\[ \beta_j = \gamma_{\beta 0} + \gamma_{\beta 1} u_j + \eta_{\beta j} \]

Error terms:

\[ \epsilon_{ij} \sim N(0, \sigma^2_y) \]

\[ \eta_{\alpha j}, \eta_{\beta j} \sim N(\gamma_{\alpha 0} + \gamma_{\alpha 1} u_j, \gamma_{\beta 0} + \gamma_{\beta 1} u_j, \sigma^2_{\alpha \beta}) \]
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Error terms:

\[ (\eta_j^\alpha, \eta_j^\beta) \sim N \left( \left( \begin{array}{c} \gamma_0^\alpha + \gamma_1^\alpha u_j \\ \gamma_0^\beta + \gamma_1^\beta u_j \end{array} \right), \left( \begin{array}{cc} \sigma_\alpha^2 & \rho \sigma_\alpha \sigma_\beta \\ \rho \sigma_\alpha \sigma_\beta & \sigma_\beta^2 \end{array} \right) \right) \]

\[ \epsilon_{ij} \sim N(0, \sigma_y^2) \]
Varying-intercept and -slope with level-2 predictor model

Scatterplot with two sections’ students in green and gold

Regression lines from unpoled regressions

HLM and unpoled lines
Varying-intercept and -slope with level-2 predictor model

Scatterplot with two sections' students in green and gold

Regression lines from unpoled regressions

HLM and unpooled lines

Scatterplot of conceptual gains by prior expected difficulty

Regression lines from multiple regression

HLM and unpooled lines

Prior expected difficulty

Conceptual gains

Curriculum experience

- Experienced
- Middle
- Now
- Non-user

Curriculum experience

- Experienced
- Middle
- Now
- Non-user
- Non-user

Non-user

Experienced

Middle

Now

Non-user

Large n

Small n

Unpooled

HLM and unpooled lines

Conceptual gains

-0.25

0.25

0.0

-0.1

0.1

0.2

0.5

Prior expected difficulty

-0.5

-0.25

0.0

0.25

0.5

2

4

6
Preliminary Study Findings

- More within-section variability than between-section variability
- Sections tended to have lower conceptual gains without using the curriculum, on average
- Students expecting the course to be difficult tended to have higher conceptual gains, except with experienced instructors
- Currently still model building
Hierarchical Models Web Application

Demo on laptop if interested
Created using shiny package in R

Cal Poly Shiny App Series (manuscript to be submitted to TISE)
- http://shiny.stat.calpoly.edu
- http://themrjw.shinyapps.io/Hierarchical_Models
Conclusion

Key points to carry away
- Hierarchical models are used to incorporate predictor variables observed from multiple sets of observational units in a single model
- The idea of (random) coefficients varying across groups
- Individual-level variables can be a function of group-level variables

Future work
- Recruit more non-users and other institutions for participation
- Continue to explore additional models
- Manuscript to be submitted to SERJ