National High School and Junior College Mathematics Club
Mu Alpha Theta (M A TH)

601 Elm Avenue, Room 423, Norman, Oklahoma 73069

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Professor Elliot A. Tanis, Chairman
Department of Mathematics
HOPE COLLEGE
Holland, Michigan 49423

Dear Professor Tanis:

"A Card Matching Problem" has traveled many miles across the country to our referees and has been selected for publication. We have delay ed answering your letter, hoping to be able to tell you it would be in the fall issue of The Math Log.

H owever the backlog of articles is too great, and it will not reach the typesetters until the next issue is being prepared. At that time you will receive a proof for correction.

Articles selected for publication, must also fit the makeup of the Log. Some fine papers have been held up because of their length; the shorter the paper, the more likely it will reach print quickly.

Many thanks for your interest in The Math Log and your patience with the lengthy process of "getting into print".

Cordially,

Richard V. Andree

Richard V. Andree

RVA:jv

U.S.A. MATH OLYMPIAD Continued

3. A random number selector can only select one of the nine integers 1,2, . . . ,9, and it makes these selections with equal probability. Determine the probability that after n selections ($n > 1$), the product of the n numbers selected will be divisible by 10.
4. Let R denote a non-negative rational number. Determine a fixed set of integers a, b, c, d, e, f such that, for *every* choice of R,

$$\left| \frac{aR^2 + bR + c}{dR^2 + eR + f} - \sqrt[3]{2} \right| < \left| R - \sqrt[3]{2} \right|$$

5. A given convex pentagon ABCDE has the property that the area of each of the five triangles ABC, BCD, CDE, DEA and EAB is unity. Show that every non-congruent pentagon with the above property has the same area, and that, furthermore, there are an infinite number of such non-congruent pentagons.

A CARD MATCHING PROBLEM

Many of you are familiar with the "matching-birthdays problem." This problem may be stated: "In a group of k people, what is the probability that at least two people have the same birthday?" The result is surprising to the uninitiated.

We modify this problem so it will be easy to estimate certain probabilities experimentally.

Each of 6 students has a standard deck of 52 playing cards. If each student draws one card at random from his deck, we shall find the probability that at least 2 of the 6 cards drawn match. Call this event A. Event A can occur in many different ways. E.g., two cards are the same and the other four cards are all different. Or, three cards are the same and the other three are all different. Or, two pairs of cards match and the other two cards are different, etc. Because event A can occur in several ways, we shall find the probability of the complement of A, namely A'. Then the probability of A is given by

$$P(A) = 1 - P(A').$$

Trial \ Person	1	2	3	4	5	6	Match
1	12	3	10	43	51	20	No
2	13	38	36	3	21	39	No
3	47	49	15	35	19	42	No
4	41	21	31	39	46	35	No
5	39	30	2	36	45	18	No
6	18	18	50	7	31	32	Yes
7	44	9	41	18	18	12	Yes
8	49	42	52	2	50	7	No
9	25	6	5	35	45	10	No
10	43	43	6	36	24	49	Yes
11	1	9	38	8	19	10	No
12	15	31	46	13	2	23	No
13	22	7	32	38	17	28	No
14	28	32	39	38	26	46	No
15	29	48	11	12	38	48	Yes
16	19	29	3	11	49	4	No
17	5	42	47	20	44	52	No
18	24	7	20	28	5	17	No
19	29	3	31	29	18	18	Yes
20	37	32	1	43	39	7	No
21	14	14	27	38	8	50	Yes
22	37	21	14	42	24	33	No
23	49	30	6	34	16	46	No
24	12	41	43	35	9	17	No

Table I

The complement of A occurs when each student draws a different card. The probability of this occurring is

$$P(A') = \frac{52}{52} \cdot \frac{51}{52} \cdot \frac{50}{52} \cdot \frac{49}{52} \cdot \frac{48}{52} \cdot \frac{47}{52} = .741$$

Note that the first student may draw any one of the 52 cards in his deck. In order that the second student's card does not match this card, he has 51 favorable choices out of the 52 cards in his deck, etc.

Thus the probability of at least one match is

$$P(A) = 1 - P(A') = .259$$

Empirically this means that if you perform such an experiment, replacing the drawn cards and shuffling the decks between trials, approximately 1/4 of the time at least one match will occur.

This experiment was simulated 24 times on an IBM 1130 computer. The 52 cards are numbered successively from 1 through 52. For this experiment 6 matches occurred in the 24 trials. See table 1.

This problem can be modified in several ways. E.g., if we are interested in matching only the value on the card and not the suits, there are 13 possible choices. The probability that the drawn cards of the 6 students do not match is then given by

$$\frac{13}{13} \cdot \frac{12}{13} \cdot \frac{11}{13} \cdot \frac{10}{13} \cdot \frac{9}{13} \cdot \frac{8}{13} = .256.$$

Thus the probability of at least one match is $1 - .256 = .744$. i.e., in several repetitions of this experiment, about 3/4 of the time a match will occur. This experiment was also simulated 24 times on the computer. For these 24 trials 17 matches occurred. See table 2.

Trial \ Person	1	2	3	4	5	6	Match
1	7	3	3	2	11	7	Yes
2	11	12	9	6	4	7	No
3	1	8	11	8	6	12	Yes
4	4	13	4	6	6	7	Yes
5	7	2	5	2	9	12	Yes
6	11	9	12	5	8	1	No
7	9	7	11	5	2	3	No
8	5	7	1	11	12	11	Yes
9	12	4	11	8	9	7	No
10	4	5	11	12	5	1	Yes
11	4	1	12	4	5	3	Yes
12	5	7	7	11	13	8	Yes
13	11	3	8	9	6	8	Yes
14	11	5	11	12	11	2	Yes
15	8	6	3	4	12	2	No
16	4	11	6	1	4	12	Yes
17	6	10	1	1	10	8	Yes
18	2	12	5	12	10	5	Yes
19	1	5	7	9	6	12	No
20	1	10	9	1	4	8	Yes
21	10	4	5	9	7	1	No
22	5	9	5	8	4	7	Yes
23	3	11	13	1	7	13	Yes
24	13	11	3	6	13	13	Yes

Table II

This problem can also be modified by using a partial deck. E.g., if each of the 6 students has a deck which contains 3 suits or 39 cards, what is the probability of a match?

You can clearly make many other modifications of this problem. For an interesting example, see the article by Richard S. Kleeber, "A Classroom Illustration of a Nonintuitive Probability", in *The Mathematics Teacher*, May, 1969.

Elliot A. Tanis, Hope College, Holland ,Michigan