Gambling, Expected Loss, and Expected Number of Plays to Lose $100

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1. INTRODUCTION

Almost every state in the USA permits some form of gambling—bingo, horse racing, dog racing, sports betting, jai alai, numbers games, casinos, or lotteries. Atlantic City opened its first casino in May of 1978 and gamblers were losing approximately $700,000 per day at this one casino.

We shall examine 3 casino games and the Michigan lottery. For each of these we shall look at expected losses for the player by considering for each game the question, "If I am willing to lose $100 by placing $1 bets, on the average, how many bets can I place?"

2. EXPECTED VALUE

An urn contains two balls numbered 1 and one ball number 4. If a player draws a single ball from the urn and the payoff to the player in the number of dollars indicated on the selected ball, then on the average the player can expect to win

\[ E = \frac{2}{3} \times \frac{1}{3} + 4 \times \frac{1}{3} = \frac{2}{3}. \]

A fair asking price for the privilege of playing this game is $2.

If the asking price or charge to play the game is \( C = \$2.10 \) per play, a player can expect to lose \( L = \$0.10 = 10\% \) on the average per play. A player who has $100 to lose can expect, on the average, to play this game \( n \) times before losing the $100, where \( n \) satisfies

\[ (\$0.10)(n) = \$100 \]

\[ n = \frac{100}{0.10} = 1000. \]

In a similar manner, if

\[
\begin{align*}
C &= \$2.08, & L &= \$0.08, & n &= 1250; \\
C &= \$2.05, & L &= \$0.05, & n &= 2000; \\
C &= \$2.02, & L &= \$0.02, & n &= 5000; \\
C &= \$2.01, & L &= \$0.01, & n &= 10000. \\
\end{align*}
\]

3. CHUCK-A-LUCK

In chuck-a-luck three dice are rolled. One possible bet is to bet on a number, say 4. Suppose that the size of the bet is $1. If one 4 is rolled, the player wins $1 and the dollar bet is returned; if two 4's are rolled, $2 is won and the dollar bet is returned; if three 4's are rolled $3 is won and the dollar bet is returned. If no 4's are rolled the bet is lost. The expected or average loss for each game played is
\[ E = \frac{3!}{111111} \left( \frac{1}{6} \right)^2 + \frac{2!}{11111} \left( \frac{1}{6} \right)^2 + \frac{3!}{1} \left( \frac{1}{6} \right)^3 - \frac{3!}{5} \left( \frac{1}{6} \right)^3 \]

\[ = -0.07871 = -7.871\% . \]

For each 100 bets of $1 the player can expect to lose

\[ 100(-0.07871) = 7.87. \]

A player who has $100 to lose can expect to place, on the average,

\[ n = \frac{100}{0.0787} = 1270 \]

$1 bets before losing the $100.

The number of trials, \( n \), to lose $100 placing $1 bets in chuck-a-luck was simulated on the computer. Ten repetitions of this simulation yielded the following values for \( n \):

\[
\begin{array}{cccc}
1486 & 620 & 1024 & 1073 \\
1490 & 1553 & 3008 & 704 \\
& & & 770
\end{array}
\]

For these 10 numbers, the average is 1293.5 which is close to 1270. Note however that \( n \) can be much smaller than 1270 or much larger than 1270.

4. ROULETTE

There are many different possible bets in roulette, all of which provide the same expected loss to the player. For example, if a bet is made on red, or on black, or on even, or on odd, the probability of winning is \( 18/38 = 0.47368 \). Note that each of these bets loses if green 0 or green 00 is the outcome.

If a bet of $1 per game is made, a player's expected (average) winnings (actually losses) per game is

\[ E = \$1(0.47368) - \$1(0.52632) \]

\[ = -0.05264 = -5.264\% . \]

Thus for each 100 bets of $1 like those mentioned above, a player can expect to lose

\[ 100(-0.05264) = 5.26. \]

A player who has $100 to lose can expect to place

\[ n = \frac{100}{0.05264} = 1900 \]

$1 bets like those above before losing $100.

5. CRAPS

In craps a pair of dice is rolled and the sum of the spots is observed. The player wins with a 7 or 11, loses with a 2, 3, or 12 on the first roll. If any other number comes up - that's the players "point" and the player continues to roll the dice. If the player's "point" comes up again before a 7 is thrown, the player wins, otherwise the player loses.
The probability of winning is 0.49293. Thus the probability of losing is 0.50707.

The expected (average) winnings (actually losses) per game for a player who places $1 bets is

\[ E = $1(0.49293) - $1(0.50707) \]
\[ = -$0.01414 = -1.414\text{.} \]

Thus, for each 100 bets of $1, a player can expect to lose

\[ 100(-0.01414) = $1.414 \text{.} \]

A player who has $100 to lose can expect to place, on the average,

\[ n = \frac{100}{0.01414} = 7072 \]

$1 bets before losing the $100.

The number of trials, \( n \), to lose $100 placing $1 bets in the game of craps was also simulated on the computer. The results of 10 such simulations yielded some interesting values for \( n \), namely:

\[
\begin{array}{cccccc}
7,360 & 6,490 & 2,100 & 2,874 & 4,550 \\
3,378 & 25,398 & 1,636 & 594 & 15,588 \\
\end{array}
\]

Note that although the average of these numbers, 6,996.8 is close to 7072, most of the time \( n \) was less than 7072 and it required the values \( n = 25,398 \) and \( n = 15,588 \) to obtain this average value of \( n \) equal to 6,996.8.

6. MICHIGAN LOTTERY

For the Michigan Lottery, we shall first consider the 50¢ Michigame tickets. These tickets are printed with a two digit number and with a three digit number. If the 2 digit number on the ticket matches the 2 digit number that is drawn, a prize of $5 is given. If the 3 digit number on the ticket matches the 3 digit number that is drawn, a prize of $50 is given. The probability of matching the 2 digit number is 1/100. The probability of matching the 3 digit number is 1/1000. Thus if these were the only possible prizes, the expected value of the game (purchase of one 50¢ ticket) would be

\[ E = ($5)(1/100) + ($50)(1/1000) - $0.50 \]
\[ = -$0.40 \text{.} \]

Thus, for each 100 bets of $1, (that is, the purchase of 200 50¢ tickets) a player can expect to lose

\[ 100(-0.080) = $80 \text{.} \]

A player who has $100 to lose can expect to purchase

\[ n = \frac{100}{0.40} = 250 \]

50¢ tickets before losing the $100.
Looking back at the urn with two $1 balls, and one $4 ball, a charge of $2.80 would make it equivalent to the Michigan Lottery if only $5 and $50 prizes were awarded. Since no one would buy Michigan lottery tickets under such circumstances they throw in prizes of $2,00 up to $100,000 or more. To qualify for the large prizes it is necessary to match both the 2 digit and the 3 digit number. The player then is eligible for TV "Super Play".

In the Daily Lottery the player selects a 3 digit number from 000 thru 999. Three different wagers are possible -- straight, 6-way box, 3-way box. For a $1 bet the payoffs are $500, $83, and $166, respectively. The expected values of this game to the player are, respectively,

\[
E_s = 500(\frac{1}{1000}) - 1 = -50\$
\]

\[
E_6 = 83(\frac{6}{1000}) - 1 = -50.2\$
\]

\[
E_3 = 166(\frac{3}{1000}) - 1 = -50.2\$
\]

That is, for each 1000 bets of $1, the player can expect to lose about $500. To put it another way, for 1000 straight bets of $1, on the average the state will return $500 in prizes. For 1000 "boxed" bets of $1, on the average the state will return $498 in prizes.

During the fiscal year 1977-78 (October 1, 1977 - September 30, 1978), gross ticket sales amounted to $326,463,000. Prize money totaling $155,231,000 was reserved. Thus for every $100 paid into the lottery for tickets,

\[
\frac{(155,231,000/326,463,000)}{(100)} = \frac{155,231}{326,463} = \frac{155231}{326463} \\
\approx 0.4755
\]

was reserved for prize money. That is, an "investment" of $100 resulted in an average loss of $52.45. The average is of course weighted heavily by the winners of large prizes so that most players average winnings per $100 would be much less than $47.55.

Through fiscal year 1978, ticket sales have amounted to $1,206,974,000. The State General Fund has received $549,253,156 and $563,372,000 has been reserved for prizes.

7. SUMMARY

Some people can afford to lose $100 while many others cannot. Some people can afford to lose much more than $100.

TIME, October 21, 1974, page 68, told about three Saudi Arabian princes who played roulette at the Monte Carlo Casino. At one point the princes were $2,000,000 ahead. They continued playing and ended up losing more than $6,000,000 which proves the old axion once formulated by Casino Founder Camille Blanc: "Money won by gamblers is just money loaned."