Using MAPLE V Release 5

To Find the p.d.f.'s of

Sums of Random Variables

Joint Mathematics Meetings

January 13-16, 1999

San Antonio, Texas

Elliot A.Tanis

Department of Mathematics
Hope College
Holland, MI 49422-9000

tanis@hope.edu

http://www.math.hope.edu/~tanis

ABSTRACT

Simulation is an effective way for students to gain understanding of the Central Limit Theorem. MAPLE can do simulations. But MAPLE brings the added capability of being able to illustrate the Central Limit Theorem theoretically.

Let Y equal the sum of n rolls of an m-sided die. It is not difficult to find the p.d.f. of Y but it can be tedious if n is very large. MAPLE can do this very easily. Graphically the probability histogram of Y can be compared to a normal p.d.f. with mean $\frac{n(m+1)}{2}$ and variance $\frac{n(m^2-1)}{12}$. Animation is an effective way to illustrate the convergence to normality as n increases.

Let Y equal the sum of a random sample of size n from a continuous uniform distribution on the interval (0, 1). MAPLE can find the p.d.f. of Y and use animation to illustrate the convergence of the distribution of Y to a normal distribution.

Now let Y equal the sum of a random sample of size n from the distribution with p.d.f.

$f(x) = \frac{3x^2}{2}$, $-1 < x < 1$. Again MAPLE can find the p.d.f. of Y. It is extremely interesting to use animation to illustrate the convergence to normality because the p.d.f. of Y has $n+1$ relative maxima. To help students understand why the p.d.f. of Y has so many relative maxima, it is instructive to look at 3-D graphs. For example, let $U = X_1 + X_2$ and let $V = X_3 + X_4$. Graph the joint p.d.f. of U and V. And then graphically illustrate the integration needed to find the p.d.f. of $U + V = X_1 + X_2 + X_3 + X_4$, the sum of four U-shaped random variables.

Sums of Discrete Random Variables

1. Let $X_1, X_2, ..., X_n$ denote the outcomes when rolling n fair dice. Let $Y_n = X_1 + X_2 + ... + X_n$ equal the sum of these n rolls. Use animation to plot probability histograms for the distribution of Y_n. Solution

2. Let $X_1, X_2, ..., X_n$ denote the outcomes when rolling n fair dice. Let $Y_n = X_1 + X_2 + ... + X_n$ equal the sum of these n rolls. Use animation to plot probability histograms for the distribution of Y_n. Superimpose a normal p.d.f. Solution

3. Let $X_1, X_2, ..., X_n$ denote a random sample of size n from the distribution with p.d.f.

$f(x) = (x+1)/6, x = 0, 1, 2$. Let $Y_n = X_1 + X_2 + ... + X_n$. Use animation to show that the probability histogram of Y_n, as n increases, becomes more and more symmetric and can be approximated with a $N(n\mu, n\sigma^2)$ p.d.f. Solution

Sums of Continuous Random Variables

Let $X_1, X_2, ..., X_n$ be a random sample of size n from a continuous distribution with p.d.f. $f(x)$, where $f(x) > 0$ for $a < x < b$ and $-\infty < a < x < b < \infty$. This section illustrates how MAPLE can be used to find the p.d.f. of the sum of these random variables.

1. Let $f(x) = 1, 0 < x < 1$. Show how to find the p.d.f. of $Y_1 = X_1 + X_2$. Solution

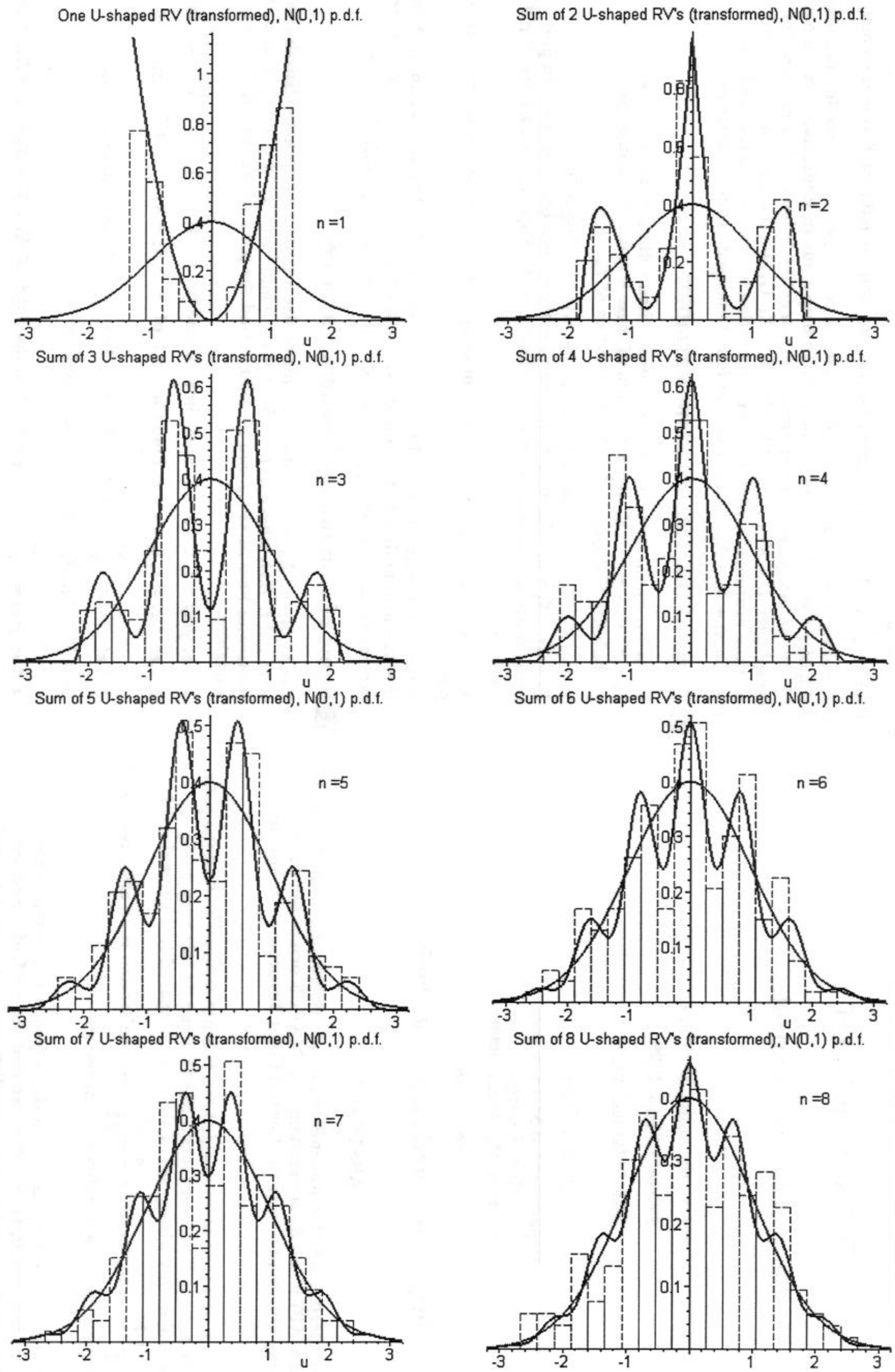
2. Let $f(x) = 1, 0 < x < 1$. Find the p.d.f. of $X_1 + X_2 + X_3 + \ldots + X_n$. Solution

3. Let $f(x) = \frac{3x^2}{2}$, $-1 < x < 1$. Show how to find the p.d.f. of $X_1 + X_2$, $X_1 + X_2 + X_3$, $X_1 + X_2 + X_3 + X_4$. Solution

4. Let $f(x) = \frac{3x^2}{2}$, $-1 < x < 1$. Find the p.d.f. of $X_1 + X_2 + X_3 + \ldots + X_n$. Solution

5. Let $f(x) = \frac{x+1}{2}$, $-1 < x < 1$. Show how to find the p.d.f. of $Y_1 = X_1 + X_2$. Solution

6. Let $f(x) = \frac{x+1}{2}$, $-1 < x < 1$. Find the p.d.f. of $X_1 + X_2 + X_3 + \ldots + X_n$. Solution


One U-shaped RV (transformed), N(0,1) p.d.f.
n =1
Sum of 2 U-shaped RV's (transformed), N(0,1) p.d.f.
n =2
Sum of 3 U-shaped RV's (transformed), N(0,1) p.d.f.
n =3
Sum of 4 U-shaped RV's (transformed), N(0,1) p.d.f.
n =4
Sum of 5 U-shaped RV's (transformed), N(0,1) p.d.f.
n =5
Sum of 6 U-shaped RV's (transformed), N(0,1) p.d.f.
n =6
Sum of 7 U-shaped RV's (transformed), N(0,1) p.d.f.
n =7
Sum of 8 U-shaped RV's (transformed), N(0,1) p.d.f.
n =8
u