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M.C. Escher and Computers

Elliot A. Tanis
Department of Mathematics
Hope College
Holland, Michigan 49423
(616) 392-5111 Ext. 3015

1. INTRODUCTION

A computer with graphics capabilities provides an excellent medium for artistic purposes. It is possible to use the computer to draw many different types of artistic designs. The author has worked with several students who have given oral reports of their 'computer art' projects at an Annual Meeting of the Michigan Section of the Mathematical Association of America [1-6]. The author also presented an hour address at one of these meetings [7]. In addition some of our work has been published [8-10].

In this paper we restrict our attention to repeating patterns or tilings of the plane. The inspiration for this study in the work of the Dutch graphics artist, Maurits C. Escher (1897-1972) who was intrigued by repeating patterns and the idea of filling the plane with figures of animals, fish, birds, etc. that leave no gaps and do not overlap. Several books contain examples of his work [11-16]. Mr. Escher was originally inspired by the Moorish mosaics in the Alhambra in Granada, Spain, and in the mosque, La Mezquita, in Córdoba.

2. REPEATING PATTERNS

An excellent mathematical discussion of repeating patterns or the plane symmetry groups is contained in an article by Doris Schattschneider [17]. For our purposes think of a repeating pattern or tessellation as a tiling of the plane. A unit cell (tile) of the pattern is a smallest region of the plane which has the property that the set of all of its images covers the plane. These
tiles, when laid in parallel rows, cover the plane.

There are four types of transformations in the plane that are necessary for studying and classifying tessellations. They are translations, rotations, reflections, and glide reflections. These will all be illustrated in the examples.

Crystallographers have developed a system for classifying crystals according to their symmetries. Repeating patterns in the plane can be classified using the two-dimensional counterpart of their system. Examples are given in [15].

It can be shown that there are 17 distinct plane symmetry groups. These are summarized in [17]. We shall give illustrations of four of these groups.

3. ROLE OF THE COMPUTER

Using a Tektronix 4051 graphics computer and the BASIC language, we have written programs that can be used to draw repeating patterns of each of the 17 symmetry groups. Most of our first examples were simulations of tessellations that were drawn by M.C. Escher. Six books in the references [11-16] contain examples of Escher’s tessellations.

We are currently in the process of adapting our programs so that they can be run on an Apple computer. Both the Tektronix 4051 and the Apple computer programs are written in BASIC. However there are differences between these two computer systems. The resolution on the Tektronix computer is far superior to that of the Apple. Colored pens can be used on a plotter that is driven by a Tektronix computer to obtain colored output. The attractiveness of an Apple computer is the dynamic output on a colored monitor. If a hard copy is desired, the image on the screen can be printed in color using a colored printer.
Although our drawings are done in color, and they will hopefully be illustrated in color during the talk, they are printed in black and white in the proceedings. Thus the reader will have to supply the color.

4. EXAMPLES

Two examples are given to illustrate each of the four transformations in the plane. For each of the four transformations, we first give a simple pattern and then an Escher-type example.

4.1 TRANSLATION

We begin with a tessellation that is based on translation only. Examples of letters of the alphabet that have no symmetry or rotation properties are F, G, J, and R. We use the letter F. You may think of the unit cell or a tile as a square with the letter F on it, similar to those used in the game of Scrabble. In Figure 1 a single tile is given. With eight similar tiles we can fill the rectangle given in Figure 2.
A tessellation of birds, patterned after an Escher drawing, is given in Figures 3 and 4. An individual tile is given in Figure 3. If 12 such tiles are combined, they yield Figure 4. Of course this could be made as large as you please provided that you have a sufficiently large number of tiles.

4.2 REFLECTION

Some letters of the alphabet have mirror lines or lines of reflection. Some of these are vertical and some are horizontal. For example A and M can be reflected across a vertical line drawn through their centers while B, D and E can be reflected across a horizontal line through their centers. We use an A turned on its side. In Figure 5 half of the letter A is drawn. This motif is then reflected across a line that runs through the center of the rectangle which forms the unit cell giving Figure 6. Six of these tiles, or unit cells, are then combined to give Figure 7. In addition to the mirror line through the center of the unit cell, the horizontal sides of the unit cell are also mirror lines.
We designed a top view of fish to form an Escher-type tessellation that is based on the property of reflection. In Figure 8 half of a fish that is headed west plus half of a fish that is headed east forms the motif. This motif is reflected across a horizontal line through the center of the unit cell, yielding Figure 9. Four of these tiles are then combined to yield Figure 10. In order to get a better overall view of these fish, they are drawn 1/4 as large in Figure 11.
4.3 ROTATION

Rotations that are possible for constructing plane symmetry patterns are called 2-fold, 3-fold, 4-fold, or 6-fold, corresponding to rotations of a motif about a point of $180^\circ$, $120^\circ$, $90^\circ$, and $60^\circ$, respectively. It would be true, for example, that at a point of 4-fold rotation, a motif would be rotated about the point four times through $90^\circ$ onto itself.

In Figure 12 a triangle is given. Take the center of the square as a point of 4-fold rotation. Rotating the triangle through $90^\circ$ and printing it four times yields Figure 13. This square then becomes the tile, or unit cell. Six of these
tiles yield Figure 14. Note that each corner of the unit cell is also a point of 4-fold rotation. Also the midpoints of the unit cell are points of 2-fold rotation.
An example based on a drawing by M.C. Escher is given in Figures 15, 16, and 17. Figure 15 gives the motif. This motif is rotated successively 90° about the center of the unit cell to complete a single tile in Figure 16. Six of these tiles yield the completed drawing in Figure 17.

4.4 GLIDE REFLECTION

For a glide reflection a motif is first translated (glided) along a line before it is reflected. In Figure 18 we begin with a letter b. If this letter b is glided to the right and then reflected about a line through the center of the unit cell, the b becomes a P as illustrated in Figure 19. Eight of these unit
cells combine to give Figure 20. Note that if the unit cell is translated to the
right a distance equal to half its length, it can then be reflected across its
lower edge onto the Pb in the bottom row. Thus we see that glide reflection
lines are the horizontal edges of the unit cell and a line through the center of
the unit cell and parallel to the horizontal edges.

An example based on a drawing by Escher is given in the remaining figures.
In Figure 21 a motif is given. This motif is translated to the right and
reflected across the center of the unit cell to yield Figure 22. Six of these
unit cells form Figure 23. Using smaller toads we have Figure 24.
5. SUMMARY OF REPEATING PATTERNS

We have written computer programs in BASIC for drawing examples of each of
the 17 possible plane symmetry groups. It is not possible in a paper of this
length to give examples of each of them. However the author is willing to send
examples of each of the 17 groups to the interested reader.

6. CONCLUSIONS

These are many ways in which the computer can be used for artistic
purposes. This paper discusses one area, that of repeating patterns. An
understanding of the mathematical properties of repeating patterns is very
useful in analyzing the patterns and developing original drawings.

The computer forms an effective interface between mathematics and art.
Projects like those given in this paper are especially appropriate for students
in a liberal arts college where students take courses in both mathematics and
art.

Microcomputers like the Apple II provide an effective and relatively
inexpensive tool for the mathematically inclined art student or the artistically
inclined mathematics student to develop some interesting designs and artistic
works.

REFERENCES

Michigan University, May, 1977.

Elliot A. Tanis


